

SIMULATED LOW-GRAVITY SLOSHING IN CYLINDRICAL TANKS INCLUDING EFFECTS OF DAMPING AND SMALL LIQUID DEPTH

by

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ABSTRACT

Liquid sloshing in cylindrical tanks is studied under conditions of simulated low gravities by using small diameter tanks to get surface tension forces comparable in magnitude to the body forces. In the tests, the effects of finite liquid depths and the determination of the smooth wall damping are emphasized. The experimental and theoretical results show that the fluid dynamics are affected by small h/d ratios in much the same way as for normal, large Bond number sloshing. Measurements of the slosh damping indicate that the damping increases as the Bond number decreases, and two correlation equations for the damping factor are proposed. An equivalent mechanical model developed previously is extended to include h/d variations and linear viscous damping. Comparisons of the force response predicted by the model to that measured in the tests verify the model to a high degree of confidence.

INTRODUCTION

The sloshing of the liquid fuel contained in a space system can strongly affect the performance of the system. During launch and powered flight, the liquid fuel is acted upon by strong body forces, but, during orbital coasting or in deep space, the body forces or "gravity" forces are reduced substantially, and the liquid motion is governed by other, primarily surface, forces. Sloshing under these conditions is usually called "low-g" sloshing or, more exactly, "low Bond number" sloshing.

Because of the lack of a convenient low gravity laboratory, not much data exist concerning low-g sloshing. Habib [1]* has reviewed most of the pertinent work done prior to 1965, and, recently, Yeh [2] and Chu [3] have studied analytically low-g sloshing in axisymmetric tanks; however, no numerical examples were given. As part of a study of the Apollo spacecraft propulsion system, a number of approximate analyses of low-g liquid motions, such as reorientation, ullage gas entrainment, and sloshing have been formulated [4], but these analyses pertain to nearly zero gravity, a regime where almost no experimental data are available for verification of the analyses. In work reported previously by us [5], an experimental and theoretical study of moderately low-g sloshing was conducted. The slosh force data given in [5] were the first such data, apparently, ever collected under low Bond number conditions

*Numbers in brackets denote references.

and, except for the present tests, are still the only existing force measurements. The experimental results were obtained by simulating low gravity through the use of small diameter tanks; in this way, the ratio of gravity forces to surface tension forces (the Bond number, NB_0) could be made equal to values of NB_0 expected in, for example, orbital coasting of Saturn-class boosters. Clark and Stephens [6] also obtained data on low-g slosh damping and natural frequency by this same method. Other experimental results have been gathered by free-fall tests in "drop towers" [7, 8]. All of these results, in general, are for specialized tank geometries or situations, and no theory has yet been able to explain completely the dynamics of low-g sloshing throughout the range from true zero-gravity (zero Bond number) to normal or high gravity (large Bond numbers).

The purpose of the work reported here was to extend the research described in [5] to include the effects of small liquid height-to-tank diameter ratios and to determine the magnitude of viscous damping under small Bond number conditions. Improvements in the experimental apparatus allowed better and more complete measurements to be made than these reported in [5]. Likewise, modifications and improvements in the theory now show that the theory is in excellent agreement throughout the range of Bond numbers tested.

EXPERIMENTAL APPARATUS AND PROCEDURE

With two major exceptions, the experimental setup used in the present tests was similar to that described in Ref. [5]. First, instead of attaching the small dynamometer package directly to the armature of an 1100-lb electromagnetic shaker, the experimental package was attached to a massive horizontal shake table which was then excited in pure translation by a much smaller, 50-lb output electromagnetic shaker. The dynamometer package (without its protective cover) is shown attached to the shake table in Fig. 1; the tank on the left, with an inverted ellipsoidal bottom, is one used in other tests. Because of the linear ball bearings guiding the shake table and the general ruggedness of the supports, an excellent sinusoidal excitation signal, with little out-of-plane motion, was obtained.* This improvement, and an improvement in the electronic amplification system of the slosh force signal, allowed sufficiently small excitation amplitudes to be used to sweep completely through the slosh resonance frequency without encountering swirling motion of the liquid. Thus, slosh damping factors could be obtained by the usual half-bandwidth technique. Second, a carbon film potentiometer was attached directly to the support frame; this allowed a continuous monitoring of the displacement amplitude with a consequent large improvement in the accuracy of the data.

The experimental procedures, calibrations, and data reductions were the same as reported previously [5]. Briefly, however, two tanks are used for each test; one tank, empty, and called the balance tank, is used to cancel the inertia of the other tank, containing the test liquid and called the active tank, so that the residual force felt by the dynamometer when the active tank is empty is very small. The sloshing force is detected by semiconductor strain gages (gage factor = 118) mounted on the tension-compression arms of the dynamometer; the output of the gages is amplified and recorded on an

*The shake table is described in [9].

oscillograph. The excitation frequency, which could be maintained to the fourth significant figure of the period (in seconds), is determined with a digital period counter.

Three different liquids (carbon tetrachloride, methanol, and acetone) were used in four different tanks (diameters: 1.36 in., 1.04 in., 0.688 in., and 0.383 in.). The variation in tank diameter and liquid properties was sufficient to cover a range of N_{BO} from 14 to 175.

TEST RESULTS

There were two main objectives of the experimental program: (1) measure the lateral slosh force for the fundamental mode as a function of the excitation frequency and amplitude, and (2) measure the slosh damping present. The parameters to be varied were the Bond number and the liquid depth.

All of the tests were run with glass tanks and reagent grade liquids. As nearly as could be determined visually, the contact angle was zero degrees for all the liquids against the tank walls, and the sloshing motion of the liquids appeared to approximate the "free edge" or no contact angle hysteresis condition very well.

Slosh Force Response and Resonant Frequency

Figures 2 through 4 show some typical force response curves for CCl_4 and methanol. The solid lines in these figures are faired curves through the experimental data. To facilitate direct comparisons, neither the force nor the frequency is nondimensionalized in any way. Note that the combination of small excitation amplitudes, very little out-of-plane motion of the shake table, and the natural slosh damping allowed complete resonance curves to be obtained; that is, no liquid swirling or rotation was evident.

As mentioned previously, the range of N_{BO} was from 175 (CCl_4 in 1.36-in. tank) to 14 (CCl_4 in 0.383-in. tank). The range of the ratio of liquid depth to tank diameter used in the tests was from 1.25 to 0.25; even larger h/d ratios were used in some tests, but these results were substantially the same as for $h/d = 1.00$ or 1.25.* Other information given in the figures includes the amplitude of tank excitation (x_0), the resonant frequency (f_1) as determined by the peak in the response curve, the slosh damping coefficient (γ_s) as determined by the half-bandwidth technique, and the wave height (δ) at resonance.

By comparing the resonant frequency, f_1 , to that calculated by theoretical results for the undamped natural frequency at large Bond numbers [i. e., $2\pi f_1 = \{3.682 (g/d) \tanh 3.682 (h/d)\}^{1/2}$], it can be seen that the resonant frequency for $N_{BO} = 175$ is slightly lower than the corresponding high-g frequency for the same h and d . As the Bond number is decreased, however, the resonant frequency increases rapidly above the high-g frequency. This is similar to the results presented in [5], which have been confirmed by Clark and

*The depth of liquid below the bottom of the curved meniscus is used in computing h/d . The average liquid depth is larger than h by an amount $0.132 \beta d$ where β is the root of $\beta^3 N_{BO} - \beta^2 - 2/3 = 0$ [5]; this is only an approximate calculation, although fairly accurate; a great amount of exact numerical work is also available [14].

Stephens [6]; however, other analytical results imply that the low Bond number natural frequency should never fall below the high-g frequency [15]. In any event, the change in natural frequency throughout the range of N_{BO} 's greater than 10 is less than about 10%.

Some slight nonlinearity is evident in the force-response curves, especially for the smaller h/d ratios or for the larger γ_s 's. Qualitatively, however, the force response even for the smallest N_{BO} of 14 is similar to ordinary large Bond number sloshing.

Slosh Damping

For each resonant force response curve, the equivalent viscous damping present in the sloshing was computed by the half-bandwidth technique. The resulting damping coefficients, γ_s (γ_s is defined as the ratio of the apparent damping to the critical damping and is equivalent to the logarithmic decrement divided by 2π), for $h/d \geq 1.0$ are shown graphically in Fig. 5. (For smaller h/d , γ_s varies with the liquid depth, but no change in damping was apparent for $h/d \geq 1.0$; this agrees with high Bond number results.) The abscissa in Fig. 5 is $N_{GA}^{-1/2}$, N_{GA} being the Galileo number, a form of the Reynolds number pertinent to high Bond number sloshing. Although N_{GA} is usually defined as $g^{1/2} R_0^{3/2} \nu^{-1}$, it is clear on both dimensional and theoretical grounds that " g " really enters by way of the natural frequency [i.e., $f_1 \propto (g/R_0)^{1/2}$ when $N_{BO} = \infty$]. Thus, for very small N_{BO} 's, the g should be eliminated from N_{GA} , and it should be redefined in terms of natural frequency; here, however, f_1 is almost proportional to $g^{1/2}$ (for $N_{BO} > 10$), so the usual definition of N_{GA} is retained.

Both experiment [10] and theory [11] have shown that γ_s is directly proportional to $N_{GA}^{-1/2}$ for large Bond number conditions. The data included in Fig. 6 also show this since a large N_{BO} corresponds to a small N_{GA} on this plot. But, for small N_{BO} (large $N_{GA}^{-1/2}$), γ_s is considerably larger than that predicted by the usual correlation equation $\gamma_s = 0.83 N_{GA}^{-1/2}$, which is valid for large N_{BO} 's. Other experimenters have also observed the increase in γ_s [6, 12]. On a purely empirical basis, Keulegan [12] and Clark and Stephens [6] concluded that γ_s should be calculated as the sum of two parts:

$$\gamma_s = \gamma_{N_{GA}} + \gamma_{N_{BO}} \quad (1)$$

where $\gamma_{N_{GA}}$ is a function only of the Galileo number and $\gamma_{N_{BO}}$ only of the Bond number; furthermore, $\gamma_{N_{BO}} \rightarrow 0$ as $N_{BO} \rightarrow \infty$. Clark and Stephens [6] were able to correlate their data (which are the $\bigcirc$ and $\square$ points in Fig. 6) in this way by using the equation

$$\gamma_s = 0.83 N_{GA}^{-1/2} + 0.096 N_{BO}^{-3/5} \quad (2)$$

which reduces to the correct relation as $N_{BO} \rightarrow \infty$ but predicts that $\gamma_s \rightarrow \infty$ as $N_{BO} \rightarrow 0$. In the range tested by them ($8 < N_{BO} < 1000$), Eq. (2) gave a very close fit to their data, although Keulegan in his work with rectangular tanks [11] found that $\gamma_{N_{BO}}$ should vary as N_{BO}^{-1} . To check Eq. (2), the present data were tested against it, as shown in Fig. 6. The correlation is fairly good although not so good as the same equation with Clark and Stephens' original data. Part

of the discrepancy may arise from the fact that the damping in [6] was based on the log decrement of the free decay of the sloshing wave, while the present damping results were based on forced response measurements; free decay tests and forced response tests are equivalent for linear systems, but this may not be true for slightly nonlinear systems such as these.

Neither Keulegan [12] nor Clark and Stephens [5] attempted an explanation of the physics behind the evident variation of γ_s with N_{BO} ; in fact, it is not apparent why γ_s should vary independently with N_{BO} since no energy dissipation is provided by surface tension forces alone. J. W. Miles [13] has, however, analyzed the damping of surface waves in tanks by using various approximations to the dissipation provided by viscosity, by diffusion from the bulk liquid to the surface and vice versa during the stretching and contracting of the free surface when it oscillates, by soluble or insoluble films or contaminants on the free surface, and by contact angle hysteresis. He proposed γ_s should be calculated as

$$\gamma_s = \gamma_{NGA}(1 + \gamma'_s) + \gamma_L \quad (3)$$

γ'_s is a parameter dependent upon surface properties and for insoluble surface films (in which the variation of the surface tension as the surface stretches is proportional to the undisturbed surface tension), it depends only on a parameter ξ :

$$\gamma'_s = \frac{\xi^2}{(\xi - 1)^2} \quad (4)$$

where

$$\xi \propto \frac{1}{N_{GA}} \left(\frac{gR_0}{f_1^2 N_{BO}} \right) \quad (5)$$

The third term, γ_L , is the contribution to the damping by contact angle hysteresis. According to Miles, both the advance and recession of the meniscus are opposed by constant forces that depend only on the material properties of the liquid-gas-tank interface. He showed that

$$\gamma_L = \frac{\kappa f(N_{BO})}{\delta} \quad (6)$$

where $f(N_{BO})$ depends only on N_{BO} , κ is the magnitude of the constant opposing force, and δ is the wave amplitude. For the present tests, γ_L should be very small (i.e., $\kappa \approx 0$) since no contact angle hysteresis was observed; furthermore, the data of [6] indicate no variation of γ_s with δ although some slight variation is evident in our results. For these reasons, a correlation of the form

$$\gamma_s = 0.83 N_{GA}^{-1/2} (1 + AN_{BO}^n) \quad (7)$$

was attempted, which is in qualitative agreement with Miles' predicted form for the damping when γ_L is neglected. Results are shown in Fig. 7. The best fit to the data was obtained with $A = 8.20$ and $n = -3/5$ so that the correlation equation is:

$$\gamma_s = 0.83 N_{GA}^{-1/2} (1 + 8.20 N_{BO}^{-3/5}) \quad (8)$$

which gives a reasonably good correlation to both the present data and the data of [6].

Equation (8) has the merit that it shows that the energy dissipation arises through the viscosity; however, neither Eqs. (8) nor (2) can be correct for $N_{BO} = 0$ since both predict that $\gamma_s \rightarrow \infty$ as $N_{BO} \rightarrow 0$ while the experimental results obtained by Salzman, et al. [7], for $N_{BO} \approx 0$ using a drop tower indicate that $\gamma_s = 3.84 N_{GA}^{-1/2}$.

For $h/d < 1.0$, the trend of the damping data is an increase in γ_s as h/d decreases. This is similar to the variation obtained for large Bond numbers, namely:

$$\gamma_s = 0.83 N_{GA}^{-1/2} \tanh 1.84 \frac{h}{R_o} \left[1 + 2 \left(1 - \frac{h}{R_o} \right) \operatorname{csch} 3.68 \frac{h}{R_o} \right]$$

However, for the smallest N_{BO} of 14, γ_s appears to decrease slightly as h/d decreases. For this reason, and because the amount of data collected is not sufficient to predict with any confidence the variation of γ_s with both h/d and N_{BO} , a correlation equation involving h/d has not been attempted.

COMPARISON OF THEORY AND EXPERIMENT

For normal and high- g conditions, an equivalent mechanical model composed of masses, springs, and dashpots gives a very good representation of the force response characteristics of sloshing. Further, it was shown in [5] that the same kind of model, even without damping (dashpots), gives a fairly good representation of low Bond number sloshing. The model developed in [5], however, was limited to $h/d > 1$, no damping, zero degree contact angle, and a "free edge" or no contact angle hysteresis condition. Thus, in this report, the same theoretical model is extended to include linear viscous damping and any value of h/d . The zero degree contact angle and no hysteresis conditions are retained since these seem to be the most practical cases. To modify the theory, it is necessary to use a velocity potential which allows for finite liquid depth; this can be done by replacing $\exp(\lambda_n z)$ in Eq. (19) of [5] by $\cosh \lambda_n z + \tanh \lambda_n h/R_o \sinh \lambda_n z$ and then proceeding as outlined in [5]. The details are not given here. The linear viscous damping is added after the fact by modifying the force response equations.

For the proposed model, consisting of one mass, m_o , attached rigidly to the tank and one mass, m_1 , attached to the tank through a spring (spring constant k_1) and dashpot (damping coefficient γ_s), the amplitude of the force response for simple harmonic excitation of frequency f is

$$F = 4\pi^2(m_o + m_1)x_o f^2 \left\{ 1 + \frac{m_1}{m_o + m_1} \left[\frac{(f/f_1)^2}{1 - (f/f_1)^2 + 2i\gamma_s(f/f_1)} \right] \right\} \quad (9)$$

where $i = \sqrt{-1}$. The parameters f_1 and m_1 ($k_1 = 4\pi^2 f_1^2 m_1$) as calculated by the present theory are shown in Figs. 8 and 9. All of the parameters are given as

multiples of the corresponding high-g quantity calculated for the same R_0 , h/d , g , and m_T ($m_T = m_0 + m_1$ is the total mass of liquid contained in the tank); for reference, these high-g quantities are

$$f_1 = \frac{1}{2\pi} \left\{ 1.841 \frac{g}{R_0} \tanh 3.682 \frac{h}{d} \right\}^{1/2}$$

$$m_1 = 0.227 m_T \left(\frac{d}{h} \right) \tanh 3.682 \frac{h}{d} \quad (10)$$

The low-g frequency and slosh mass for $h/d = 1.0$ shown in the figures differ somewhat from the results presented in [5]; the difference is caused by retaining more terms here in the infinite series used to compute the model parameters.

By using the figures to calculate f_1 , m_1 , k_1 , and $m_0 = m_T - m$, the force response for any N_{BO} and h/d can be determined. Comparisons of the force response predicted by the mechanical model to our experimental results are shown in Figs. 10, 11, and 12; the value of γ_s used in Eq. (9) to compute the force corresponds to the experimental tests for the indicated x_0 , R_0 , h/d , and liquid. The comparison throughout the N_{BO} and h/d range is uniformly good, as can be seen. The darkened triangle ($\blacktriangledown$) along the frequency axis in each plot is the theoretical undamped natural frequency, f_1 , whereas the peak in the resonance curve locates the damped resonant frequency; the difference between the two is entirely due to the damping.

Considering the good correlation between the frequency of the theoretical peak force and the experimental peak, it may be concluded that the curves in Fig. 8 adequately predict the low-g slosh frequency. Likewise, since the peak force for the theory and experiment are very close, the slosh mass m_1 is given adequately by Fig. 9.* Thus, the proposed mechanical model gives a good representation of the low-g sloshing dynamics.

CONCLUSIONS

The experimental tests have verified that the use of small diameter tanks is adequate to simulate moderately low Bond number sloshing, including h/d variations and the effects of damping.

The smooth-wall damping coefficient was shown to increase as the Bond number decreased; for $N_{BO} > 10$, an adequate correlation of the damping coefficient is provided by either

$$\gamma_s = 0.83 N_{GA}^{-1/2} + 0.096 N_{BO}^{-1/2}$$

or

$$\gamma_s = 0.83 N_{GA}^{-1/2} \left(1 + 8.20 N_{BO}^{-3/5} \right)$$

The second equation has the virtue of being in qualitative agreement with

*The peak force depends almost entirely on only m_1 and γ_s .

existing theories, but neither correlation equation can be extended much below $N_{BO} = 10$.

The experiments, in conjunction with the theory, show that the low Bond number slosh mass, natural frequency, and spring constant all decrease more slowly as h/d decreases than do the corresponding high Bond number quantities. In other words, if the low Bond number parameters decreased at the same rate as did the high Bond number parameters, all of the frequency curves in Fig. 8, for example, would be parallel, and, in fact, all the curves would collapse onto the $h/d = 1.0$ curve. Since the smaller h/d curves are translated upward and moreover spread apart as N_{BO} decreases, it can be concluded that the frequency decreases less slowly with h/d than does $\tanh 3.682 h/d$, which is the rate of decrease for large Bond number conditions.

Comparisons of the force response predicted by the theoretical model with the actual test values verify the mechanical model to about the same degree of confidence as similar models for high Bond number sloshing. The comparisons also show the importance of accounting for the damping in making natural frequency determinations; for example, with CCl_4 in a 0.383-in. diameter tank, the actual resonant frequency is about 9.7 cps (for $h/d = 1.0$), whereas the undamped theoretical natural frequency is 9.95 cps. (The resonant frequency for the mechanical model is less than the natural frequency by the factor $\sqrt{1-2\gamma_s^2}$.)

ACKNOWLEDGEMENTS

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NOMENCLATURE

Symbol

- d - diameter of tank
- f - frequency of tank excitation
- f_1 - natural frequency or resonant frequency of sloshing
- g - acceleration of gravity or equivalent linear acceleration
- h - depth of liquid below bottom of meniscus
- k_1 - spring constant in mechanical model
- m_1 - slosh mass in mechanical model
- m_o - rigidly attached mass in mechanical model
- m_T - $m_o + m_1$, total liquid mass
- N_{BO} - Bond number, $\rho g R_o^2 / T$ where ρ and T are liquid density and surface tension.
- N_{GA} - Galileo number, $(g R_o^3 / \nu^2)^{1/2}$

- R_0 - tank radius
- x_0 - amplitude of tank excitation
- γ_s - slosh damping coefficient
- δ - amplitude of slosh wave
- ν - kinematic viscosity

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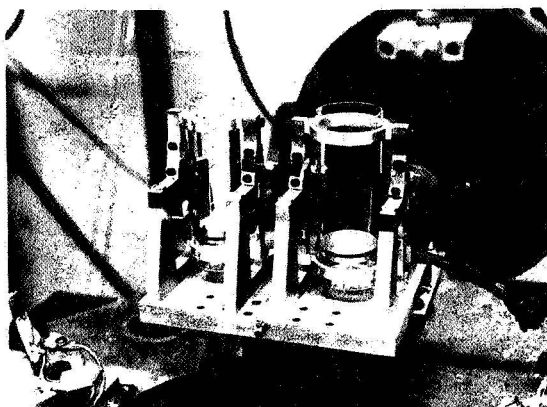


FIG. 1. Dynamometer Package on Shake Table

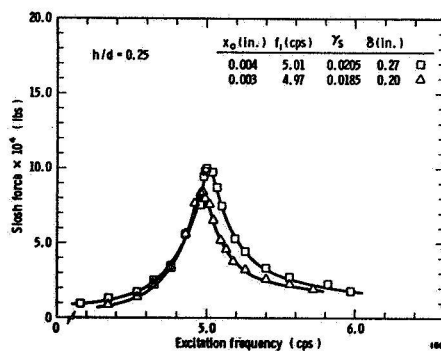


FIG. 2. Response Curves for Methanol in 1.04" Diameter Tank, $N_{BO} = 60$

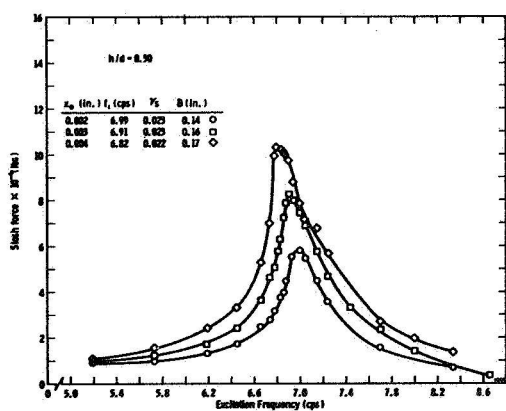


FIG. 3. Response Curves for CCl_4 in 0.688" Diameter Tank, $N_{BO} = 45$

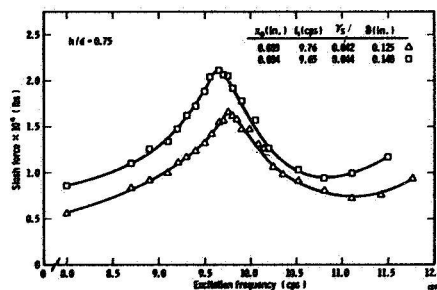


FIG. 4. Response Curves for CCl_4 in 0.383" Diameter Tank, $N_{BO} = 14$

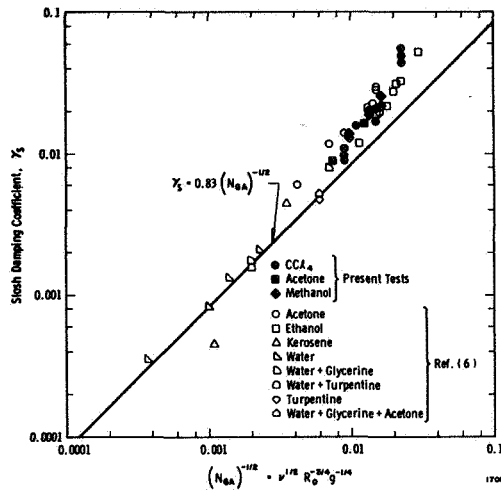


FIG. 5. Variation of Smooth Wall Damping Coefficient with Galileo Number

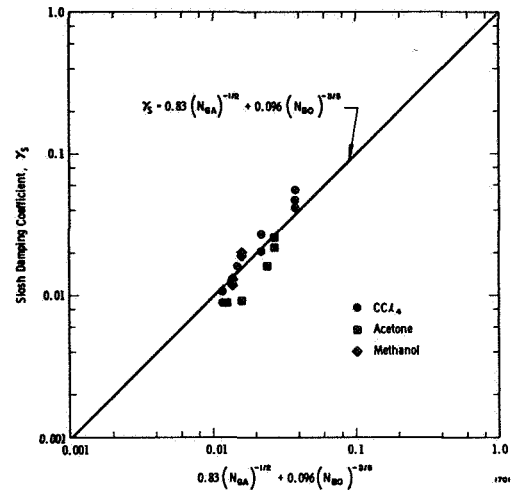


FIG. 6. Variation of γ_s with Correlation Equation of Clark and Stephens [6]

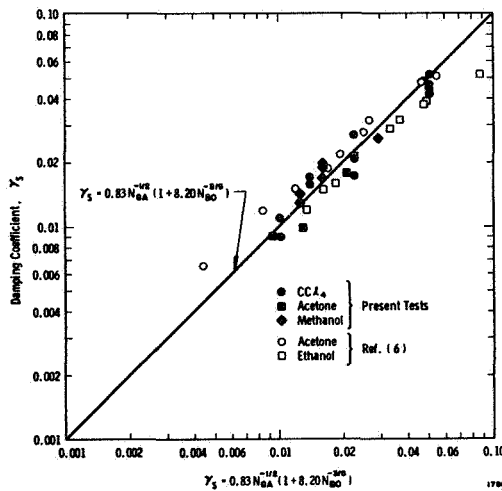


FIG. 7. Variation of γ_s with Present Correlation Equation

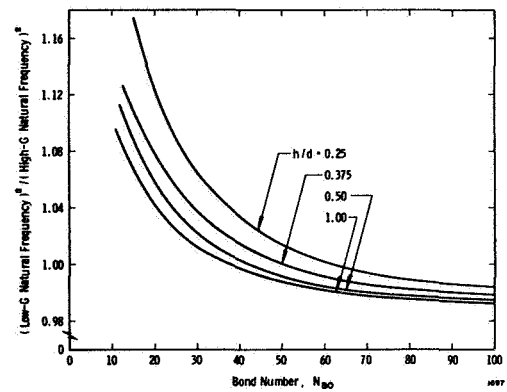


FIG. 8. Variation of Slosh Natural Frequency with Bond Number

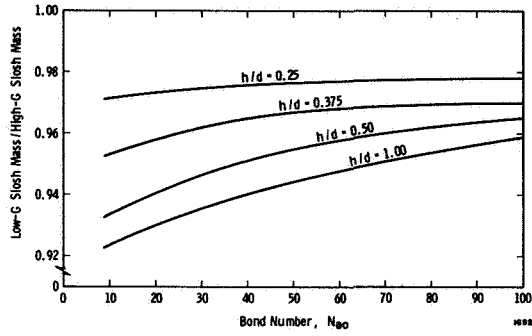


FIG. 9. Variation of Slosh Mass with Bond Number

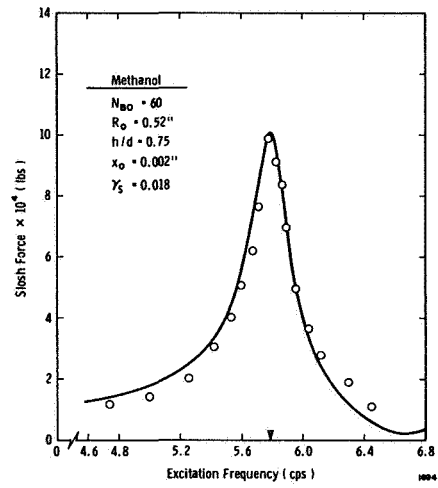


FIG. 10. Comparison of Theory and Experiment, $N_{BO} = 60$

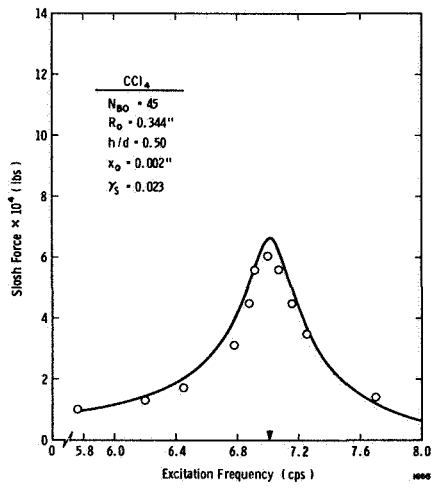


FIG. 11. Comparison of Theory and Experiment, $N_{BO} = 45$

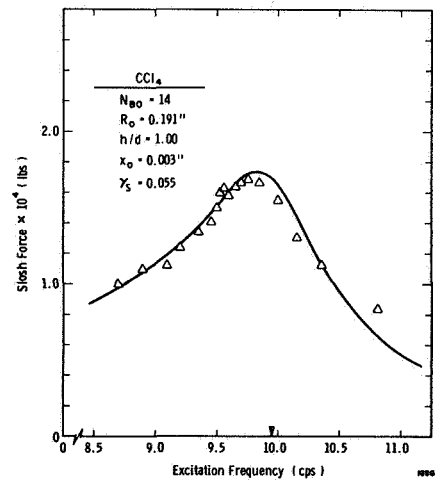


FIG. 12. Comparison of Theory and Experiment, $N_{BO} = 14$